Is Chess solvable?

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1 Introduction

Chess is a fun game to play, but for some people it is more than just a game. Professional chess players make a living of it. Big companies, like IBM, develop supercomputers to beat human players at chess. For hundreds of years, people have been fascinated by the game. The rules are simple and easy to learn, but achieving a high level of playing skill takes years of training and experience.

To be successful at chess, it is important to find good moves. During a game, the players are trying to successively find the best move in every position. But is there such a thing as the "best" move? Maybe there are some equally good moves. The German chess master Emanual Lasker even argued that there are as many best moves as there are different opponents. This inevitably leads to the question whether chess is solvable.

A game is called solvable if it is possible to determine the outcome before even playing. For instance, Tic-Tac-Toe is a solved game. If both sides make only perfect moves, the game will always end in a draw. In this easy case it is even possible to draw a tree for all possible moves. For every position, the end of the tree shows the possible outcomes for the possible moves. The players just need to choose the moves leading to desirable outcomes.

Chess is a full information game. The players know the position of all the pieces and, in principle, can calculate all possible outcomes. From this point of view, chess is solvable. But in contrast to Tic-Tac-Toe, the tree that describes chess has to be huge. There is extensive literature about chess opening, dealing with just the first few moves of the game. This paper raises the question, whether it is practically possible to solve chess.

2 Discussion

In a chess game, White always begins. Black can only react to the threats that White poses. For this reason, if chess can be solved, the only possible outcomes are "White wins" and "Draw". The only scenarios where having to move is bad are Zugzwang scenarios. They occur only in endgames and we assume that the effect is neglegible. The most rigorous way to solve chess would be to draw a tree for all possible positions. Then, starting from any position, a player would be able to make only perfect moves by following the branches and looking at the end of the tree for the outcome of each choice.

There are 12 distinct pieces in a chess game: King, Queen, Rook, Bishop, Knight and Pawn, each in black and white. On a chess board with 64 squares, there are $64!/52! \approx 1.57 \cdot 10^{21}$ possibilities to place those pieces. This is not taking into account illegal and symmetric positions, but there are many more positions with more pieces, less pieces and different piece combinations. The total number of possible positions is estimated to be $\approx 2.28 \cdot 10^{46}$[1]. This number is so high that it is practically impossible to write down all the positions.

One step into the direction of "drawing trees" are the Tablebases[2]. This is a collection of endgame positions, all possible moves for these and the outcome. Up to now, all positions with 6 pieces or less have been solved, and the number is growing. Some pieces more might be feasible, but Tablebases will never include all 32 pieces of a chess game.

Most of the possible positions will look like complete rubbish to an experienced player. And for most positions, it will be obvious who wins, because of one player having an undeniable advantage. It might be enough to collect all important positions, even though this number will still be huge.

Right now, the only way to coming closer to solving chess seems to be a combination of raw computer power and clever algorithms and heuristics. Chess engines are already able to beat the best human players of the world. With faster computers it is possible to increase the search depth, the number of successive moves the programme calculates in advance. Nowadays, typical search depths are 15 and more halfmoves, far more than the average human player is able to handle. Maybe quantum computing will significantly increase the search depth further one day.

The weak spot of chess engines is positional evaluation, but with better algorithms and heuristics the evaluations are becoming more accurate. Additionally, there is still a lot of unexplored potential in the direction of intelligent, self-learning engines.

Combining computer power and heuristics with extensive tablebases leads to engines that are able to "solve" simple positions. The definition of simple in this context has changed dramatically in the past, and will change in the future. The solution to more complex positions, positions that are closer to the starting position, will become available.

3 Conclusion

Even though solving chess for every possible position is not possible, it might be possible to develop a chess engine that reliably finds the route to victory (or a draw) through a combination of computational power and heuristics. Whether this will be realised for the starting position some day is an open question.

[1] http://de.wikipedia.org/wiki/Schach, accessed July 28, 2010
[2] http://www.k4it.de/index.php?topic=egtb, accessed July 28, 2010

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